# Introduction to Scientific Computing, PSCB57, Fall 2016
# Assignment 5
# Differential equations

- The deadline for this assignment is Friday, November 11th, 5pm.
- You must submit the assignment electronically at `http://rein.utsc.utoronto.ca/submit/`. The username is `pscb57`, the password is `2016`.
- Make sure your code passes all tests before submitting it. Your code needs to pass all tests in 3 seconds or less. If your code does not pass the automated tests, you will most likely not get any points for it.
- This assignment comes in multiple parts. Submit all your answers in one file.
- Only submit the functions you wrote, i.e. no print statements, tests or data files.
- Do not use any packages or libraries except where stated.
- The entire submission has to be 90 lines of code or less (including comments).
- You must be present at the tutorial in the following week to take a quiz. If you do not show up or fail to pass the quiz, your assignment will be marked as 0% even if it was correct.

## Part 1

Implement the Euler and midpoint methods for solving arbitrary ordinary differential equations of the form

$$\frac{\partial x(t)}{\partial t} = F(t, x(t)) \tag{1}$$

with initial conditions $x_0 = x(t_0)$. Your implementation should work for arbitrary dimensions, e.g. consider $x$ to be a vector (a list of floats). The two functions that you have to implement should have the prototype: `integrateEuler(F,x0,t0,h,N)` and `integrateMidpoint(F,x0,t0,h,N)`. The variables have the following meaning:

**`F(t,x)`** Python function, right hand side of Equation 1. Returns a list of floats.

**`x0`** Initial conditions at time $t_0$ (a list of floats).

**`t0`** Initial time (a float).

**`h`** Stepsize (a float).

**`N`** Number of steps to take (an integer).

Your functions should return two lists. First, a list of times. Second a list with the values of $x$ as you step through the integration (a list of lists). The lists should include the initial conditions.

## Part 2

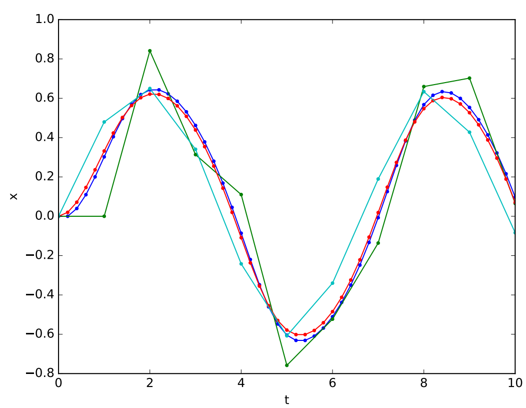
You now use the two algorithms from Part 1 to integrate the one dimensional non-autonomous differential equation

$$\frac{\partial x}{\partial t} = -x^3 - x + \sin t \tag{2}$$

with the initial condition $x(0) = 0$. Integrate the equation from $t = 0$ to $t = 10$ four times.

1. Use the Euler method and $N = 10$ steps.
2. Use the Euler method and $N = 50$ steps.
3. Use the midpoint method and $N = 10$ steps.
4. Use the midpoint method and $N = 50$ steps.

Show all four integrations in one plot and save it as a pdf file called `ode.pdf`. Add labels $t$ and $x$ to the axes, show the individual datapoints as well as lines connecting the datapoints. The following plot shows a correct solution.




For this question, you have to use two packages. First, import the `sin` function:

```
from math import sin
```

Then, import the plotting library `matplotlib`. If you use your own computer, you might have to install the library first. When importing, please using the following two import statements. This ensures that the code will run on a machine without a graphical user interface.

```
import matplotlib; matplotlib.use("pdf")
import matplotlib.pyplot as plt
```

Then, you can create and save a plot with the following commands:

```
ax = plt.subplot()
...  your plotting commands here ...
plt.savefig("ode.pdf", format="pdf")
```